# Ricardo's Model of Comparative Cost

## About Physical Masses Moving in Linked Economic Systems

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**Abstract**

The concept of comparative advantage, developed by D. Ricardo[4] in 1817, has the important result that even when a nation does not have an absolute advantage in the production of particular goods or services with another nation, it can still be beneficial for that nation to engage in international trade. – Unlike the traditional approach that uses labor cost and productivity to introduce Ricardo's revolutionary concept, the present paper chooses the two products rice and wheat as example and focuses on physical mass of these cereals that is moved and traded between two nations. The study shows that obeying Ricardo's concept triggers a maximum of mass flow inside the total system called "trading nations". This outcome might be helpful when issues like "sustainable development" and "sustainable economic growth" are discussed.

## 1 Theory of comparative cost

In his theory of comparative cost D. Ricardo[4] starts out from the idea that trade takes place when production technologies are different. He shows that trade is advantageous for all involved countries. Even a country that lags behind in using modern technology can benefit from trade. And, a country with highly developed industry must not fear the competition with countries that are characterized by low wages. At present Ricardo's results are regarded as important basis of international trade theories[3].

In this paper we will adopt Ricardo's original example and introduce two countries which manufacture two products that need one factor of production. As a rule this shall be manpower. The model is in equilibrium state in which all markets are competing with each other without restriction. We define that manufactured products can not be distinguished, neither by country nor by manufacturer. Transports don't incur costs; and human labor shall be totally interchangeable inside a country. However, as a rule human labor shall be linked with different productivity. In other words, we assume that production technology in both countries is different. Whereas human labor can be transferred among industries of one country without incurring costs, this shall not be possible between countries. Furthermore, unemployment shall not exist and consumers maximize their benefit with regard to income.

### 1．1 Rice and wheat as an example

We introduce two countries called HOME and FOREIGN as Suranovic[6] did in his paper. Both countries shall produce rice (R) and wheat (W) respectively. The functions of production of both goods can be written as follows.

$$m_{Rh} = \frac{L_{Rh}}{a_{LRh}} \tag{1}$$

$$m_{Wh} = \frac{L_{Wh}}{a_{LWh}} \tag{2}$$

$$m_{Rf} = \frac{L_{Rf}}{a_{LRf}} \tag{3}$$

$$m_{Wf} = \frac{L_{Wf}}{a_{LWf}} \tag{4}$$

Constants and variables have the meaning shown.

$m_{Rh}, m_{Wh}, m_{Rf}, m_{Wf}$ : Mass of rice and wheat in HOME and FOREIGN expressed as [kg]

$L_{Rh}, L_{Wh}, L_{Rf}, L_{Wf}$ : Labor needed in both countries to produce rice and wheat expressed as [ persons × time ]

$a_{LRh}, a_{LWh}, a_{LRf}, a_{LWf}$ : specific labor requirement to produce rice and wheat in both countries expressed as [ persons × time/kg ]

The amount of labor that can be used in both countries is limited as expressed by the equations

$$L_{Rh} + L_{Wh} = L_h\ , \tag{5}$$

$$L_{Rf} + L_{Wf} = L_f\ . \tag{6}$$

### 1．2 Production possibility frontier (PPF)

By combining the four functions of production (1)－(4) and the two limitations of labor (5), (6) the production possibility frontier (PPF) can be written as shown below.

$$a_{LRh} \cdot m_{Rh} + a_{LWh} \cdot m_{Wh} = L_h$$

$$a_{LRf} \cdot m_{Rf} + a_{LWf} \cdot m_{Wf} = L_f$$

Rearrangement results in

$$m_{Wh} = \frac{L_h}{a_{LWh}} - \left(\frac{a_{LRh}}{a_{LWh}}\right) \cdot m_{Rh}\ , \tag{7}$$

$$m_{Wf} = \frac{L_f}{a_{LWf}} - \left(\frac{a_{LRf}}{a_{LWf}}\right) \cdot m_{Rf}\ . \tag{8}$$

Both functions (7) and (8) respectively describe the dependence of wheat production from the amount of rice produced in both countries.

### 1.3 Absolute advantage

A country has an absolute advantage regarding the production of a specific product over another country, if the product can be produced with higher productivity. When talking about absolute advantage comparison is not limited to national borders. In case of an absolute advantage of HOME compared to FOREIGN concerning the production of rice the following relations apply.

$$a_{LRh} < a_{LRf}\ , \tag{9}$$

$$\frac{1}{a_{LRh}} > \frac{1}{a_{LRf}}\ . \tag{10}$$

Expression (9) means that workers in HOME require less working hours for production of rice [person × time / kg] than workers in FOREIGN need. And expression (10) shows that labor productivity [kg / (person × time)] of HOME concerning rice is higher compared to FOREIGN.

### 1.4 Opportunity cost

Opportunity costs are understood as costs that arise when passing up the next best choice regarding the use of a scarce factor[2]. In our case HOME and FOREIGN are free to produce rice or wheat. If one country decides to increase the production of rice it has to reduce the production of wheat because human labor is limited. The reduction of production of wheat that is necessary to increase the production of rice is expressed by the opportunity costs. These opportunity costs are represented by the slope $-(a_{LR}/a_{LW})$ of the production possibility frontier (PPF). In other words, the slope indicates how much wheat has to be given up in order to get 1 kg of rice. Accordingly, the reciprocal $-(a_{LW}/a_{LR})$ represents the opportunity costs of producing wheat compared to rice.

### 1.5 Relative advantage

A relative or comparative production advantage exists, if a country is able to manufacture a good with lower opportunity costs compared to another country. Therefore, HOME has a comparative advantage over FOREIGN if the relation

$$\frac{a_{LRh}}{a_{LWh}} < \frac{a_{LRf}}{a_{LWf}} \tag{11}$$

holds true. This relation says that HOME has to give up less wheat to produce an additional kilogram of rice than FOREIGN has. In other words, the slope of the production possibility frontier of HOME must be gentler compared to FOREIGN. If relation (11) is rearranged as follows

$$\frac{a_{LWf}}{a_{LRf}} < \frac{a_{LWh}}{a_{LRh}}\,, \tag{12}$$

it says that FOREIGN is able to produce wheat with lower opportunity costs than HOME does. Therefore, FOREIGN has a comparative advantage regarding production of wheat. If this is the case, HOME must have a comparative advantage regarding production of rice. In fact, this is what relation (11) already said.

## 2 Consequences of trade

Literature[3, 6] reports that on specific conditions two countries will be better off when giving up autarkical provision of goods for the benefit of trade. In the following paragraphs these conditions will be studied in detail.

### 2.1 Situation in cases of autarky and trade under condition of limited labor

Under autarkical conditions HOME and FOREIGN are compelled to produce rice and wheat by themselves. In this case (indicated by subscript $a$) the amount of rice and wheat can be written as

$$(m_{Rh})_a = \frac{(L_{Rh})_a}{a_{LRh}}\,,$$
$$(m_{Wh})_a = \frac{(L_{Wh})_a}{a_{LWh}}\,,$$
$$(m_{Rf})_a = \frac{(L_{Rf})_a}{a_{LRf}}\,,$$
$$(m_{Wf})_a = \frac{(L_{Wf})_a}{a_{LWf}}\,.$$

In addition, HOME shall have a comparative advantage over FOREIGN concerning production of rice; i. e.

$$\frac{a_{LRh}}{a_{LWh}} < \frac{a_{LRf}}{a_{LWf}}\,.$$

When HOME and FOREIGN start trading with each other, HOME will give up the production of wheat in favor of importing the grain from FOREIGN. At the same time FOREIGN will give up the production of rice and import respective amounts from HOME. In case of trade (indicated by subscrip $t$) the produced amounts of rice and wheat can be expressed as

$$(m_{Rh})_t = \frac{(L_h)_t}{a_{LRh}}\,,$$
$$(m_{Wh})_t = 0\,,$$
$$(m_{Rf})_t = 0\,,$$
$$(m_{Wf})_t = \frac{(L_f)_t}{a_{LRf}}\,.$$

In both countries labor is a scarce factor of production. We define available labor $L$ to be identical in both countries as well as in case of autarky and trade:

$$(L_h)_t = (L_h)_a \ ,$$

$$(L_f)_t = (L_f)_a \ .$$

Identical conditions shall also be valid in both countries regarding the specific requirement of labor and the production of rice and wheat:

$$(a_{LRh})_t = (a_{LRh})_a = a_{LRh} \ , \tag{13}$$

$$(a_{LWh})_t = (a_{LWh})_a = a_{LWh} \ , \tag{14}$$

$$(a_{LRf})_t = (a_{LRf})_a = a_{LRf} \ , \tag{15}$$

$$(a_{LWf})_t = (a_{LWf})_a = a_{LWf} \ . \tag{16}$$

The version of Ricardo's model of comparative advantage explained in literature shows that the total amount of produced goods (i. e. rice and wheat in our case) compared to autarky will increase when trade relations are established between HOME and FOREIGN. Therefore, we can regard HOME and FOREIGN as subsystems of a total system comprised of both countries' economies. The amount in kilogram of rice produced in the total system in case of autarky and trade can be expressed as follows.

$$(m_{Rs})_a = (m_{Rh})_a + (m_{Rf})_a \ ,$$

$$(m_{Rs})_a = \frac{(L_h)_a}{a_{LRh}} - \left(\frac{a_{LWh}}{a_{LRh}}\right) \cdot (m_{Wh})_a + \frac{(L_f)_a}{a_{LRf}} - \left(\frac{a_{LWf}}{a_{LRf}}\right) \cdot (m_{Wf})_a \ ,$$

$$(m_{Rs})_t = \frac{(L_h)_t}{a_{LRh}} \ .$$

The change of mass $\Delta m_{Rs}$ of rice in the total system before and after establishing trade relations is calculated as

$$\Delta m_{Rs} = (m_{Rs})_t - (m_{Rs})_a \ ,$$

$$\Delta m_{Rs} = \frac{(L_h)_t}{a_{LRh}} - \left\{ \frac{(L_h)_a}{a_{LRh}} - \left(\frac{a_{LWh}}{a_{LRh}}\right) \cdot (m_{Wh})_a + \frac{(L_f)_a}{a_{LRf}} - \left(\frac{a_{LWf}}{a_{LRf}}\right) \cdot (m_{Wf})_a \right\} \ .$$

Rearrangement of this expression in consideration of relations (13)－(16) results in

$$\Delta m_{Rs} = \left(\frac{a_{LWh}}{a_{LRh}}\right) \cdot (m_{Wh})_a - \left\{ \frac{(L_f)_a}{a_{LRf}} - \left(\frac{a_{LWf}}{a_{LRf}}\right) \cdot (m_{Wf})_a \right\} \ . \tag{17}$$

By taking

$$\left(\frac{a_{LWh}}{a_{LRh}}\right) \cdot (m_{Wh})_a = \frac{(L_h)_a}{a_{LRh}} - (m_{Rh})_a$$

and

$$\left\{ \frac{(L_f)_a}{a_{LRf}} - \left(\frac{a_{LWf}}{a_{LRf}}\right) \cdot (m_{Wf})_a \right\} = (m_{Rf})_a$$

into consideration and substituting respective terms in equation (17), we finally get

$$\Delta m_{Rs} = \frac{(L_h)_a}{a_{LRh}} - \left[(m_{Rh})_a + (m_{Rf})_a\right] \ .$$

If condition

$$\frac{(L_h)_a}{a_{LRh}} > \left[(m_{Rh})_a + (m_{Rf})_a\right]$$

is valid, the difference will be positive; i. e. more rice is available in the total system in case of trade than it is in case of autarky. In the same way the relation for the amount of wheat produced in the total system can be derived. This difference is

$$\Delta m_{Ws} = \frac{(L_f)_a}{a_{LWf}} - \left[(m_{Wf})_a + (m_{Wh})_a\right] \ .$$

What has been concluded about rice can also be said about wheat. Trade leads to a higher amount of wheat in the total system compared to autarky, if the condition

$$\frac{(L_f)_a}{a_{LWf}} > \left[(m_{Wf})_a + (m_{Wh})_a\right]$$

holds true.

### 2．2 Availability of goods as system function

Ricardo's model gives a description of borderline cases. In order to perform a detailed study we will continue to treat HOME and FOREIGN as subsystems of a total system. The total mass $m_s$ of rice and wheat available in this system is calculated as sum of the respective products in both subsystems:

$$m_s = m_{Rh} + m_{Wh} + m_{Rf} + m_{Wf} \ .$$

The amount of wheat produced in both subsystems can be expressed by the amount of rice using the functions（7）and（8）which represent the respective production possibility frontiers.

$$m_s = m_{Rh} + \frac{L_h}{a_{LWh}} - \left(\frac{a_{LRh}}{a_{LWh}}\right) \cdot m_{Rh} + m_{Rf} + \frac{L_f}{a_{LWf}} - \left(\frac{a_{LRf}}{a_{LWf}}\right) \cdot m_{Rf}$$

or

$$m_s = \left(\frac{L_h}{a_{LWh}} + \frac{L_f}{a_{LWf}}\right) + \left(1 - \frac{a_{LRh}}{a_{LWh}}\right) \cdot m_{Rh} + \left(1 - \frac{a_{LRf}}{a_{LWf}}\right) \cdot m_{Rf}$$

Concerning the establishment of trade relations in order to enforce division of labor based on the concept of comparative advantage inequation (11) has decisive importance.

$$\frac{a_{LRh}}{a_{LWh}} < \frac{a_{LRf}}{a_{LWf}}$$

To investigate gradually the influence of this relation on the decision for trade or autarky we define the four variables $a_{LRh}$, $a_{LWh}$, $a_{LRf}$ and $a_{LWf}$ to be a function of the sweeping variable $i$ on the condition $0 \leq i \leq 9$ as follows:

$$(a_{LRh})_i = i + 1 \quad , \quad (a_{LWh})_i = 10 - i \quad , \quad (a_{LRf})_i = 10 - i \quad , \quad (a_{LWf})_i = i + 1$$

These definitions give a continuous curve depending on the the sweeping variable shown by figure 1.

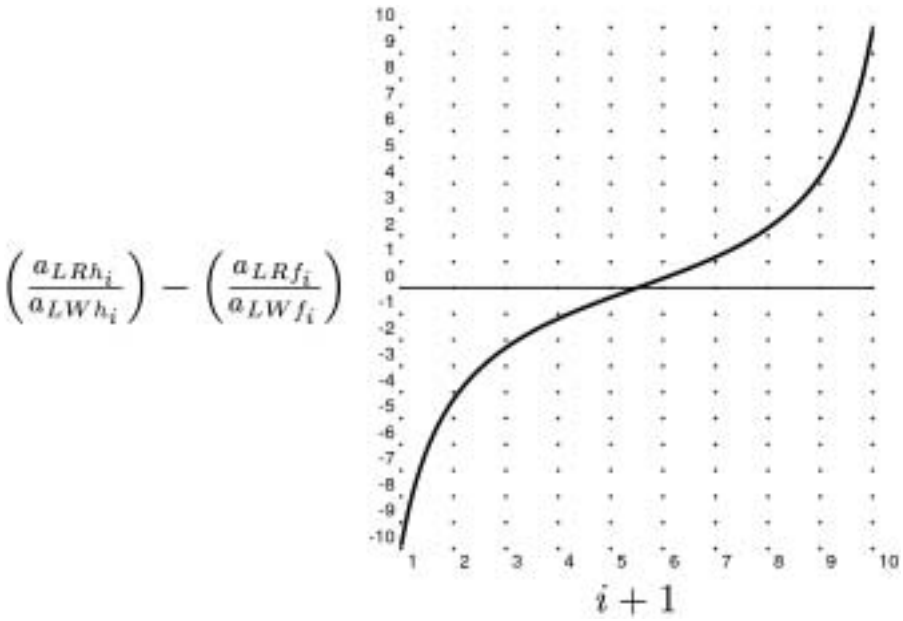


Figure 1: Difference of specific labor requirement quotients

In addition to varying the specific labor requirement we change the amount $m_{Rh}$ and $m_{Rf}$ of rice produced in subsystems HOME and FOREIGN from 0 to maximum in opposed manner in order to investigate the influence on the total system. However, one must not overlook that the respective maximum value is not a constant, but depends on $a_{LRh}$ and $a_{LRf}$. Both functions of the sweeping variable $i$ are determined by the quotients

$$(R_{LRh.max})_i = \frac{L_h}{(a_{LRh})_i} \quad \text{and} \quad (R_{LRf.max})_i = \frac{L_f}{(a_{LRf})_i} .$$

To facilitate numerical calculation, we will define the working week to be $L_h$=48 and $L_f$=48 hours respectively. Furthermore, we introduce a second sweeping variable $j$ to let $m_{Rh}$ and $m_{Rf}$ change in the range $0 \leq j \leq 10$ as follows:

$$(m_{Rh})_{i,j} = (m_{Rh.max})_i \cdot \frac{j}{10} \quad \text{and} \quad (m_{Rf})_{i,j} = (m_{Rf.max})_i \cdot \left(1 - \frac{j}{10}\right) .$$

The result of this variation calculation is shown in figure 2 as topogram and figure 3 as 3D picture.

Under the condition $i = 0$ (cf abscissa in figure 2) HOME has a comparative advantage with respect or rice. Therefore, Ricardo's model recommends to cease production of rice in FOREIGN and switch to exclusive production of wheat. Because, in case of $j$=10 (cf Y-axis in figure 2) we get

$$(m_{Rf.max})_{i=0,j} \cdot \left(1 - \frac{j}{10}\right) = 0 .$$

This corresponds to the coordinate in the upper left corner of figure 2. In this case the total mass $m_s$ reaches a maximum. If the figures for the respective specific demand of labor reverse according to figure 1, exclusive production of rice in FOREIGN has to be favored. In this

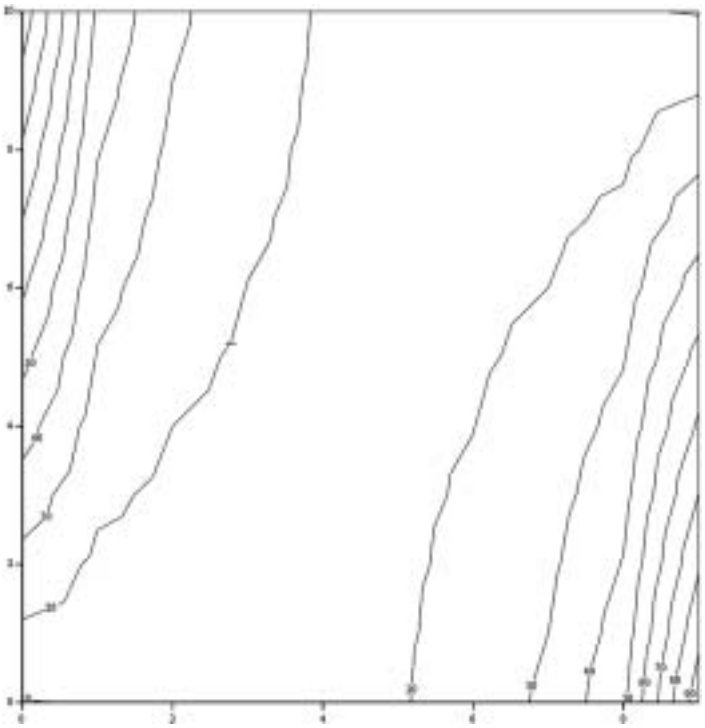

Figure 2: Isoquanta diagram of the total system's mass $m_s$

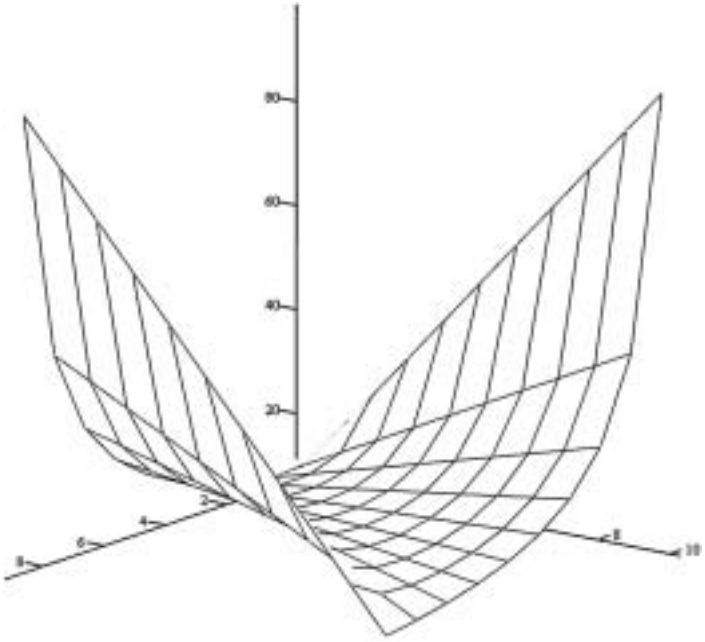


Figure 3: 3D-plot of system mass $m_s$

case the system function again reaches a maximum (cf lower right corner in figure 2).

## 3 Conclusion

Whenever Ricardo's model of comparative cost is cited, it is done to verify the advantage of trade versus autarky. The model emphasizes that even trading nations with different level of technology will benefit from a win-win position compared to autarky.

The present investigation of two subsystems producing rice and wheat respectively has shown again that a country with a comparative advantage concerning the production of a specific good will be better-off when it concentrates all efforts on the production of this good while stopping production of the other good that has no comparative advantage. The reason for this observation is extreme specialization under the condition of constant labor productivity as well as limited availability of labor. However, this result is achieved at the expense of increasing physically moved masses of rice and wheat which reaches a maximum in the total system.

When pleading for international trade, Ricardo's model is welcomed as an example of a typical "win-win" situation[1, 6]. However, one must not fail to see that the word "winner" implies the existence of a loser. Therefore, if two winners exist, as the expression "win-win" suggests, there must be a big loser. Indeed, as the present study reveals, the "win-win" situation of both trading nations leads to an increased amount of physical masses, which causes damage to the environment addressed by the concepts of "Ecological Footprint"[8], "Factor 4"[7], and "Factor 10"[5]. However, at present the evaluation of environmental impacts caused by moving masses is still an issue of debate among specialists.

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